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# POINT-BASED MODELS FOR COMPENSATION OF THERMAL EFFECTS IN DIMENSIONAL METROLOGY

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## ABSTRACT

*In a hybrid approach to geometric modelling and metrology, a computational model of an object (component or system) is formed and modified in the light of the known external conditions such as temperature and loading. Measurements of the physical component are taken and compared with the predictions obtained from finite element analysis. There are thus several models of the object: an accurate geometric CAD model; a point-based finite element mesh model prediction of distortion involving many points; and measurement results from a small number of points on the physical object. The research question is whether the information from these models can be combined to assist in down-stream activities such as assembly of components. The aim is to modify the mesh model continuously so that it agrees with the metrology results at the measured points. Each measured deformation corresponds to a rigid-body transform. The methodology is to treat the measured points as forming a polyhedron and to interpolate the local transform smoothly along the edges, across the faces, and away from the faces. The results from some examples, involving large distortions, are presented.*

## KEYWORDS

Model variation, thermal variation modelling, geometric algebra, metrology.

## 1. INTRODUCTION

Dimensional metrology has many sources of uncertainty, one of the most significant being temperature. For this reason, the standard dimensional metrology temperature is defined to be 20 degrees Celsius [4]. For large-scale assemblies, temperature control can be difficult so these are particularly prone to thermal effects. These effects include: thermal expansion of the measurand (the object being measured); and variation in the refractive index of air when optical instrumentation is used [11]. Particularly in large structures, significant thermal gradients can be present. Some indoor environments may experience gradients of 3-5 degrees Celsius in any direction, often predominantly vertically. Temperatures are also not static. Throughout the day, the temperature is likely to vary by several degrees and this can influence manufacturing operations. Temperature is considered to be one of the critical factors in the success of assembly processes [2].

Thermal compensation in metrology is generally attempted by simply scaling linearly, based upon coefficients of thermal expansion and often a single ambient temperature reading. This is of limited benefit. It assumes that temperature is uniform within (and around) the measurand, which it is not. For assembly operations, this is also limited. For example, it is far more helpful to be aware of the point-wise coordinate displacements at particular assembly fixtures or interfaces. A novel hybrid metrology method is being created in which the combination of more comprehensive temperature measurement and simulation allows predictions to be made at all parts of a component, a tooling structure, or a product assembly [32]. To improve simulation predictions, adaptations to the geometry of the measurand are required.

The conventional approach to the prediction of thermal distortion is the use of finite element analysis. The mesh model used for the analysis is typically derived from a geometric model created using a CAD system. A number of techniques are available for measuring physical objects. These include photogrammetry and laser scanning. Typically these provide accurate locations (in world space) for specific points on the physical structure. Thus there are a number of models in use. There are: the full nominal geometry described by the geometric model from the CAD system; the point-based mesh model of the distorted geometry derived from the finite element analysis and this often involves many thousands of nodes; and the physical distortion model based on a small number of measured points.

The accuracy of the distorted mesh-model is only as good as the assumptions made by the finite element approach and the boundary conditions supplied by the user. The physical distortion model is highly accurate but only applies to a small number of individual points. The research question is whether it is possible to combine the information from the various models to provide information useful for down-stream activities such as assembly of components. The question is answered in this paper by using the physical distortion measurements to modify the mesh model so that the nodes corresponding to measured points have the measured distortion. The aim is to generate a rigid-body transform at each point in space which provides the appropriate transform at each measured point to "correct" the finite element prediction. If this transform varies smoothly (over space) then it can be applied to all the other points in the mesh model to "correct" their predictions of the distortion. What results is a point-based model of the thermally distorted object which agrees with the measured at distortion at the points where physical measurement has taken place. Naturally this point-based model does not have all the geometric detail of the original CAD model; it is not intended to. However it gives a clearer indication of the true distortion of the full structure which can be used to inform subsequent activity such as assembly operations.

This paper investigates how the smoothly varying rigid-body transform can be constructed. The methodology is to regard the points where physical measurement takes place as forming a polyhedron in space. The required transform is derived at the vertices of the polyhedron from the measurement results. It is then extended to the whole of three-dimensional space by interpolation, firstly along the edges of the polyhedron, then across its faces, and finally away from the faces. results are obtained from examples in which a metal frame framework is thermally distorted and finite element analysis used to model the process. It is shown that the deformations along the sides of the frame can be obtained from information about how the corners distort.

Section 2 reviews the literature in the areas of compensation for thermal effects in metrology and of modelling of rigid-body transformations. The idea of combining predicted distortions and physical measurements is part of a hybrid approach to metrology and compensation which is discussed in section 3. Section 4 introduces the numerical scheme for establishing the correcting transform across an object Some examples are given in section 5. At this stage, the approach is still being developed and investigated, and the results seem encouraging. Conclusions are presented in section 6.

## 2. LITERATURE SURVEY

Uncertainty evaluation of measurement with thermal variation can be challenging, further complicated by variation in thermal expansion coefficients in materials [37]. Various studies have considered measurement and compensation for thermal effects in different applications. One example is machining processes where the temperatures resulting from operations are usually significantly above the ambient room temperature [40]. This can result in poorly finished components especially if the specifications are demanding. In some cases, it is possible to compensate for machine tool errors arising from thermal effects in real time [13].

It is of course possible to measure the temperature of an object and its environment in many ways (and with various levels of accuracy) [31, 33]. These allow monitoring of what is happening in an assembly environment. There is then the possibility of using physical temperature measurements to inform thermal variation models and hence make appropriate compensation of thermal effects.

Assessing the effect of temperature on the tolerance stack-up of assemblies is of importance in product design [21, 22]. Various methods and software packages are available to perform tolerance analysis on assemblies [5]. A study of assemblies in the automotive industry suggested that simulations of assembly variation should be combined with studies of thermal expansion, rather than considering these two issues separately[24]. These methods are all typically used in the design phase, rather than being brought into the execution of manufacturing processes, which would confer the advantage of early warning when products are straying from specification.

For large objects, an entirely different range of errors can be induced by gravitational effects. A body may distort under its own weight and the distortion varies according to how the object is orientated and supported. Again there are implications for assembly operations and stack up of the tolerances [25].

Measurement techniques, such as photogrammetry and laser trackers, are available to try to determine where distortions occur in a body. However during assembly it can be time-consuming to continually perform checks. However these may only be point-based. Various computational procedures are also available to simulate what distortions occur. Methods based on finite elements are common [25]. Currently a hybrid approach [32] is being considered. This attempts to bring together the results of physical measurements of an object and mesh-based simulation of how it can distort. An overview is given in section 3. Using the physical measurements (assumed to be sufficiently accurate) can allow modifications to be made to the mesh model (which are naturally subject to modelling errors).

Corrections to the point-based mesh model need to involve forms of geometric transforms and those which represent rigid-body movement preserve the local geometry. There are several ways of handling geometric transforms [30] of which the use of 4×4 matrices and homogeneous coordinates is perhaps the most commonly used. Recently interest has been renewed in the techniques of geometric algebra deriving from the seminal work of Grassmann [17, 18], Hamilton [19], Clifford [7,38], and others in the 1800's. Shoemake used quaternions to represent rotations about an axis through the origin [36]. These idea have been extended to dual and double quaternions [1, 29]. These approaches have been applied in areas such as: robotics [16], folding operations [39], and mechanisms [14].

Geometric algebra provides an environment which extends the quaternions and provides a natural representation of three-dimensional space and so allows geometry to be represented and transformed [35]. Several forms of geometric algebra have been proposed [6, 15, 27, 34]. Although the constructions are different, their representations of geometry and transforms are very similar. They have been used for various applications including: robotic vision [3], free-form motions [9], and quantum systems [28].

## 3. HYBRID APROACH

A hybrid metrology system can be thought of as one in which measurements of various physical quantities of a component and its environment are made and used to inform and update computational models of the component [32]. The precise details of such a system depend of course upon which quantities are measured physically and simulated computationally. Figure 1 shows a typical form of the proposed approach.

Key is a geometric model of the component itself with its nominal dimensions. This can be created within an appropriate CAD environment. It provides basic geometric details for various analysis packages. The figure suggest analysis in the forms of: thermal effects; distortion under loading (including self-loading); and assessment of errors in manufacture and/or tolerances in assemblies. Finite element software can deal with thermal and loading effects; specific packages are available for considering tolerance build-up [10]. Other analysis could include: modelling of fatigue damage [12], and the strength of composite structures [23]. Note that some of this analysis work depends upon results obtained from measuring the environment in which the real component has been placed. An obvious example here is the need to know the ambient temperature when undertaking thermal modelling.

The results of the analysis are used to modify the nominal geometric model so as to produce a better prediction of the state of the actual component. Figure 1 describes this deformed model as “continuous” meaning that modifications are applied to the whole model.

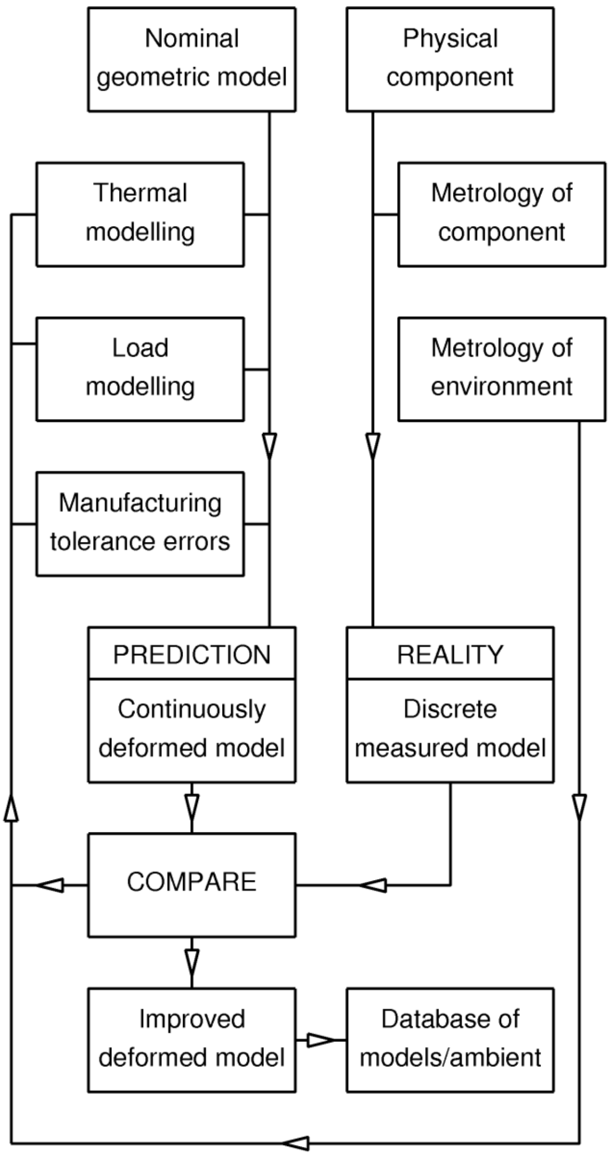


**Figure 1** Hybrid computational/metrology approach

The accuracy of this prediction is limited by whatever assumptions are made in setting up the analysis models and in their methods of solution. So the other main part of the hybrid approach is the use of metrology techniques on the real physical component. This can include: measurement of features on the component (perhaps with photogrammetry or a coordinate measuring machine), and the measurement of temperature on the component (perhaps using thermocouples) [33]. Typically, these measurements are at discrete points on the component.

There is then the opportunity to compare the predicted model with the discrete measured results. This of course requires the ability to find the appropriate discrete points in the continuous geometric model and hence determine the associated predicted values there. If there are discrepancies, then there are (at least) two possible courses of action. One is to feedback the comparison and revisit the analysis stages imposing modified solution conditions. The other is to try to update the predicted model directly so that there is agreement between it and the measured results. If this is done continuously and the original discrepancies are small, it is assumed that this brings the rest of the model into closer agreement with the physical component. It is this updating approach that is the motivation for what is considered in this paper.

Once sufficiently good agreement has been obtained between the deformed geometric model and the physical measurements, then it is possible to store the model along with the relevant environmental parameters. This allows the predicted model to be recovered if those external parameters are encountered again. It also provides good starting points for subsequent analyses when the environmental conditions have changed.

One of the intended application areas for the hybrid approach is one in which a company is concerned with assembling large structures. These are likely to deform under changes in the ambient temperature and under their own weight (depending on how they are supported). It is important to be able to know whether assembly is no longer possible (perhaps because the tolerance build-up is no longer favourable), or whether it can still be achieved if the assembly strategy is modified. It is therefore important to have good geometric models of the components of the assembly for given external conditions. It is assumed that the external conditions vary slowly (over minutes rather than seconds). This means that it is possible to use the hybrid approach continuously to predict and update the geometric model since the times required to undertake with the analysis parts are not a critical consideration.

## 4. MODELLING DISTORTIONS

As indicated previously, the underlying idea is as follows. There is a point-based mesh model (usually obtained from finite element analysis) of an object which predicts the effects of disturbances due to thermal (and possibly other) effects. This is assumed to be accurate but may suffer from the limitations inherent in the modelling used. There is also information of the actual geometric distortion at a number of discrete “measurement” points on the physical object. These are assumed to be accurate. The interest is in modifying the mesh model so that it coincides with the physical measurements at the measured points.

To investigate what can be done, this idea is initially abstracted as follows. It is assumed that there is a point-based model of an object. At a number of discrete measurement points, a rigid-body transform is specified, corresponding to the required correction. The aim is to define a function providing a rigid-body transform at all points in space which coincides with the specified transforms at the measurement points. [Note that the approach is to distort the points in a point-based model. There is no attempt here to distort the original geometric model of the object (as perhaps defined with a CAD systems). So there are no issues of trying to preserve any B-rep or CSG structures.]

Use is made of the ideas of geometric algebra of which there are several formulations [6, 15, 26]. Any of these works appropriately: it is the $\mathcal{G}_4$ given in [9, 26] which is used for the examples given here. Within the algebra, rigid-body transforms can be represented as elements of even grade, and the algebra allows such elements to be combined additively (as well as multiplicatively). The attraction of this approach is that rotations and translations are represented and manipulated in a common form.

The measurement points at which the transforms are specified are taken to be the vertices of a polyhedron in space. The interest is in extending the given transforms at the discrete points so as to obtain a transform at any point in space. This interpolated transform needs to agree with the transform at each measurement point.

It seems desirable that, at a point on a line joining two measurement points, the interpolated transform is simply an interpolation between the transforms at those measurement points. Similarly, if a point lies within the plane defined by three measurement points, then it is desirable that the interpolated transform depends only on those three measured transforms.

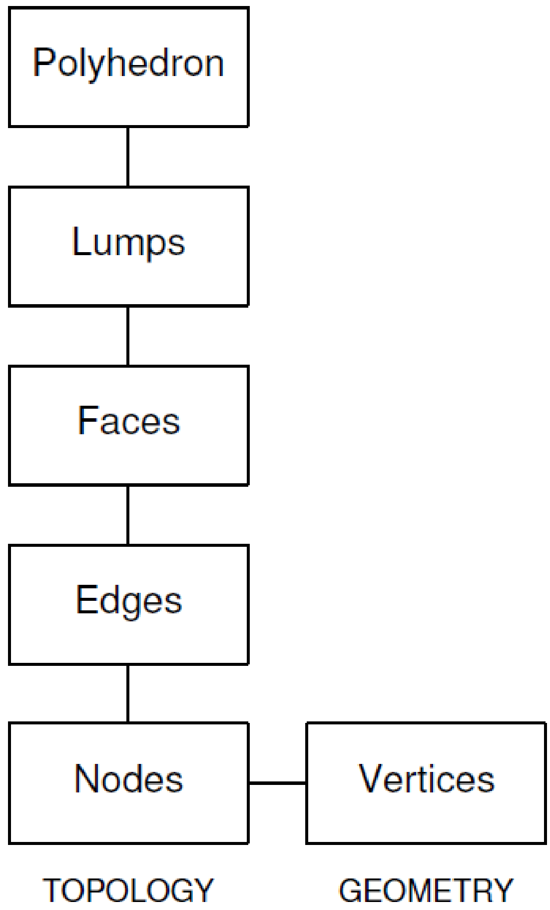


**Figure 2** Data structure for polyhedron

However such desires cannot always be fulfilled since there may be contradictions inherent in the measured transforms. For example, if four measurement points happen to lie in a plane, then there is no reason to expect that interpolating separately along the two diagonals can lead to the same result at the intersection of those diagonals. Similarly interpolating for the plane using three out of the four measurement points is unlikely to give a result that is independent of which point is omitted. Hence the previous desires are restricted to lines and planes which are part of the polyhedron defined on the measurement points.

**Relation: "member of" $\in$**

If $V$ is a vertex and $N$ is a node,

$$V \in N \iff V = N$$

If $V$ is a vertex and $E$ is an edge,

$$V \in E \iff V \text{ is an end-point of } E$$

If $x$ is an entity and $F$ is a face,

$$x \in F \iff \begin{cases} \text{EITHER} & x \text{ is an edge of } F \\ \text{OR} & x \in E \text{ for some edge } E \in F \end{cases}$$

If $x$ is an entity and $L$ is a lump,

$$x \in L \iff \begin{cases} \text{EITHER} & x \text{ is a face of } L \\ \text{OR} & x \in F \text{ for some face } F \in L \end{cases}$$

**Figure 3** Relation: "member of"

**Relation: "within" $\prec$**

For point $\mathbf{r}$ and node $N$,

$$\mathbf{r} \prec N \iff ||\mathbf{r} - N|| = 0$$

For point $\mathbf{r}$ and edge $E$,

$$\mathbf{r} \prec E \iff \mathbf{r} \text{ lies on } E \text{ between its end-points}$$

For point $\mathbf{r}$ and face $F$,

$$\mathbf{r} \prec F \iff \mathbf{r} \text{ lies on plane of } F \text{ and within boundary of } F$$

For point $\mathbf{r}$ and lump $L$,

$$\mathrm{r} \prec L \iff \mathbf{r} \text{ lies in the region of space defined by } L$$

**Figure 4** Relation: "within"

**Unsigned distances**

$$\mathrm{dist}(\mathbf{r}, N) = ||\mathbf{r} - N|| = ||\mathbf{r} - \mathrm{proj}(\mathbf{r}, N)||$$

$$\mathrm{dist}(\mathbf{r}, E) = \begin{cases} ||\mathbf{r} - \mathrm{proj}(\mathbf{r}, E)|| & \text{if } \mathrm{proj}(\mathbf{r}, E) \prec E \\ \min_{V \in E} [\mathrm{dist}(\mathbf{r}, V, E] & \text{otherwise} \end{cases}$$

$$\mathrm{dist}(\mathbf{r}, F) = \begin{cases} ||\mathbf{r} - \mathrm{proj}(\mathbf{r}, F)|| & \text{if } \mathrm{proj}(\mathbf{r}, F) \prec F \\ \min_{E \in F} [\mathrm{dist}(\mathbf{r}, E, F)] & \text{otherwise} \end{cases}$$

**Figure 5** Distance functions

**Weights**

$$\mathrm{wnode}(V, \mathbf{r}, N) = \begin{cases} 1 & \text{if } V \in N \text{ and } ||\mathbf{r} - V|| = 0 \\ 0 & \text{if } V \in N \text{ and } ||\mathbf{r} - V|| \neq 0 \\ 0 & \text{if } V \notin N \end{cases}$$

$$\mathrm{wedge}(V, \mathbf{r}, E) = \begin{cases} \dfrac{\sum_{N \in E} \mathrm{wnode}(V, N, \mathrm{proj}(\mathbf{r}, N))/\mathrm{dist}(\mathbf{r}, N)}{\sum_{N \in E} 1/\mathrm{dist}(\mathbf{r}, N)} & \text{if } V \in E \\ 0 & \text{if } V \notin E \end{cases}$$

$$\mathrm{wface}(V, \mathbf{r}, F) = \begin{cases} \dfrac{\sum_{E \in F} \mathrm{wedge}(V, E, \mathrm{proj}(\mathbf{r}, E))/\mathrm{dist}(\mathbf{r}, E)}{\sum_{E \in F} 1/\mathrm{dist}(\mathbf{r}, E)} & \text{if } V \in F \\ 0 & \text{if } V \notin F \end{cases}$$

$$\mathrm{wlump}(V, \mathbf{r}, L) = \begin{cases} \dfrac{\sum_{F \in L} \mathrm{wface}(V, F, \mathrm{proj}(\mathbf{r}, F))/\mathrm{dist}(\mathbf{r}, F)}{\sum_{F \in L} 1/\mathrm{dist}(\mathbf{r}, F)} & \text{if } V \in L \\ 0 & \text{if } V \notin L \end{cases}$$

**Figure 6** Weighting functions

## 4.1. Polyhedron structure

The measurement points define a polyhedron. This is set up as a data structure representing its elements. This is a simplified version of the structure used by the ACIS geometric modeller [8]. [However note that, as stated before, it is point-based model that is distorted. The data structure here is simply a convenient way of representing the geometry of the polyhedron and setting up the required interpolation of the transforms.]

The structure is summarized in figure 2. At the base of the structure is a list of *vertices*. Each vertex is associated with a position vector in three-dimensional space. The vertices hold the "geometry" of the object. For simplicity, the distinction between a vertex and its position vector is blurred and the two ideas are treated as the same.

The "topology" of the object starts with a collection of *nodes*. Each node is associated with a vertex. Again the distinction between a node and its vertex and position vector is blurred in what follows. An *edge* of the object comprises a pair of nodes.

A *face* of the object is an ordered collection of edges which are those bounding the face going in anticlockwise order when viewed from outside the object. The edges are coplanar and the face is a planar facet of the object. There is thus an implied orientation for the edges and the order of the nodes within each edge is such that the first node of an edge is the same as the second node of the edge that precedes it around the face. This also means that every "real" edge appears twice – once in each direction (this corresponds to the ACIS concept of a co-edge [8).

A *lump* is a collection of faces which bound a polyhedron with planar faces.

The polyhedron is used as the basis for defining weighting functions as discussed in the next subsection.

## 4.2. Weighting functions

A relation "is a member of" indicates whether a given geometric item from the polyhedral structure is part of another item in the structure. Figure 3 gives the definitions for the various cases.

A second relation is defined. This is "within" and is defined in figure 4. It is used to indicate how a general point in space relates to geometric items in the structure of the polyhedron.

A number of projection functions are created. These given the nearest point on an object to a given point in space.

If **r** is a point and $N$ is a node, then proj(**r**,$N$) is simply the point at the vertex $N$, no matter what **r** is.

If **r** is a point and $E$ is an edge, then proj(**r**,$E$) is the point on the edge (possibly extended) nearest to the point.

If **r** is a point and $F$ is a face, then proj(**r**,$F$) is the point in (the plane of) the face nearest to the point.

These projections allow the distance from a point to a geometric object to be defined. These definitions are given in figure 5.

These, in turn, allow a number of weighting functions to be defined. These are shown in figure 6. Each is defined for a given vertex $V$ of the polyhedron and a three-dimensional point **r**. These are related to other items in the polyhedral structure.

Note that the definitions involve the reciprocal of the distance function. The distance becomes zero when the point **r** lies on a geometric item in the polyhedron and care is required in carrying out the evaluation. In these cases, an infinite term appears in the numerator and denominator: these dominate the other terms involved and they themselves cancel out. This allows the effect of other geometric items to be ignored when the point lies on one particular item. This helps to ensure that the interpolated transform function behaves as required with respect to the given transforms at the measured points.

## 4.3. Interpolated transform

Even-grade elements of the geometric algebra can be assigned to the vertices. These generate the given rigid-body transforms at the measured points. Let S($V$) be the transforming element at vertex $V$.

Then, if **r** is any point in space, the following defines an even-grade element, and hence a transform, at that point.

$$\mathrm{S}(\mathbf{r}) = \textstyle\sum_{V \text{ in } L} \mathrm{wlump}(V, \mathbf{r}, L)\, \mathrm{S}(V) \qquad (1)$$

It is also possible to achieve "morphing" from one set of measured transforms to another: the first set can be identity transform to represent the state before any change is made. For general morphing, two even-grade elements $S_0(V)$ and $S_1(V)$ are assigned to each vertex. Then for a given value of a parameter $t$ ($0 \leq t \leq 1$), set

$$S(V) = (1-t)S_0(V) + tS_1(V) \tag{2}$$

Now equation (1) defines an even-grade element, $S(\mathbf{r})$, at any point $\mathbf{r}$ which varies smoothly with $t$.

## 5. EXAMPLES

The difficulty of presenting examples here is the fact that in the intended application the transforms involved are small, and so there is little to see. So three examples are provided in which the transforms are large enough that their effects are visible. The first two examples are simple test structures. The third example derives the "measured" transforms from the results of a finite element analysis of a frame structure.

### 5.1. Example 1

This first example is based on a block of size 3×2×1 units. It is represented as a set of unit cubes as shown in the top part of figure 7.

Eight "measurement" transforms are applied – one at each of the outermost corner vertices. Four of these (on the right back vertical face in the figure) act to keep the vertex in the same plane but move it in towards the centre of the face. The other four (on the left front vertical face) perform the same movement but also act to pull the face forward and to rotate it about its centre.

Thus the overall effect is to twist and stretch the block in one direction and reduce its cross-section in planes normal to this direction. The other parts of figure 7 show the effect of morphing using equation (2) with $t$ being 0.25, 0.5, 0.75 and 1.0. The last part therefore shows the effect of applying the full interpolated transform.

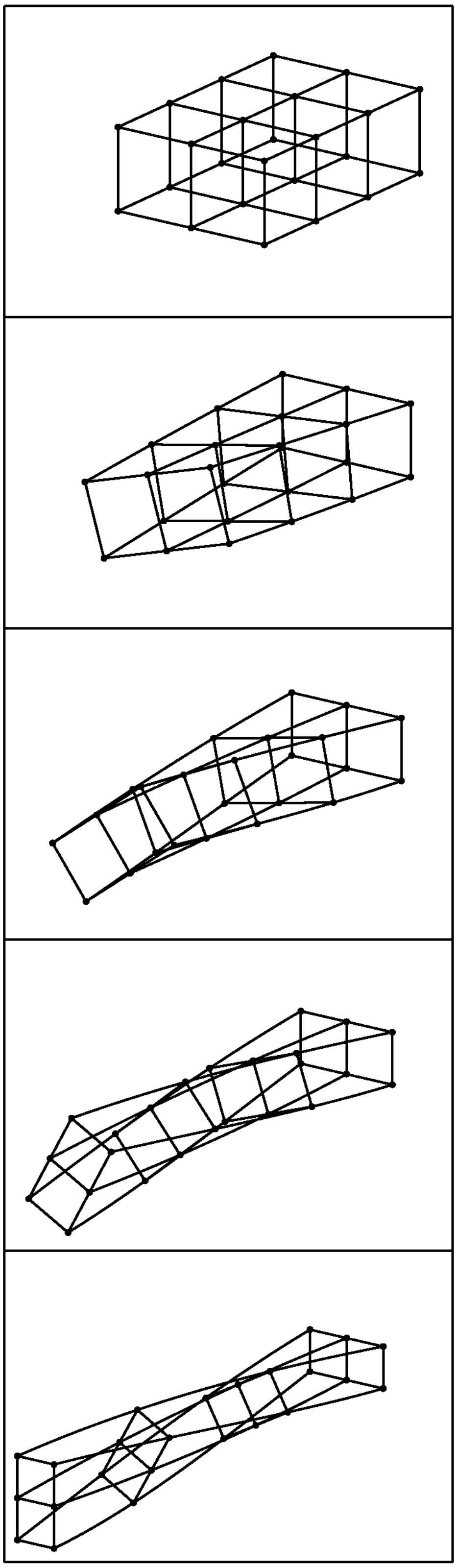

**Figure 7** Twisting and stretching of a single block

## 5.2. Example 2

Here the object being deformed is an L-shaped block as seen in the top part of figure 8. As before, measurement transforms are applied at eight vertices. Four of these are the outer vertices of the back right vertical face in the figure. These applied transforms are all the identity so that these vertices (and their face) is fixed.

The other four vertices are those of the face at the other end of the L-shape. The measured transforms here are all the same and act to rotate the face about a horizontal line through its centre and then to translate the face downwards. Since they are the same, the end-face is simply moved and not itself distorted.

The eight measurement points form the polyhedron. This is shown with the thicker lines in the upper part of figure 8.

The lines in the figure are made up of a number of small segments and the interpolated transform is applied to all their end points. The other parts of figure 8 show the effects of morphing in equal steps of the parameter. The final part shows the full effect of the interpolated transform. The back face remains fixed and the other end-face moves as expected. The other points move so that a smooth deformation is obtained. This is true even though most of the points being transformed lie outside the polyhedron of the measurement points.

## 5.3. Example 3

This last example is based on deformations arising from a finite element analysis of a simple cuboidal framework structure.

The analysis considered the effect on the structure of applying loads to the two long lower members and heat to one side. The results are shown in figure 9, with the deformation magnified. In fact the cause of the deformation is not relevant here: the analysis is simply a source of deformations on a component.

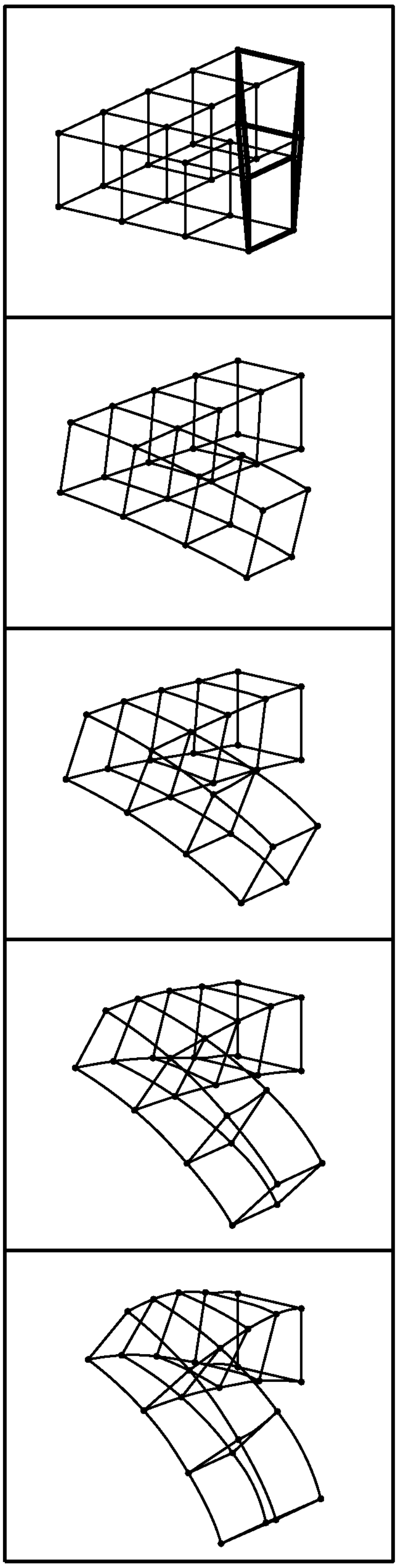

**Figure 8** Twisting of an L-shaped block

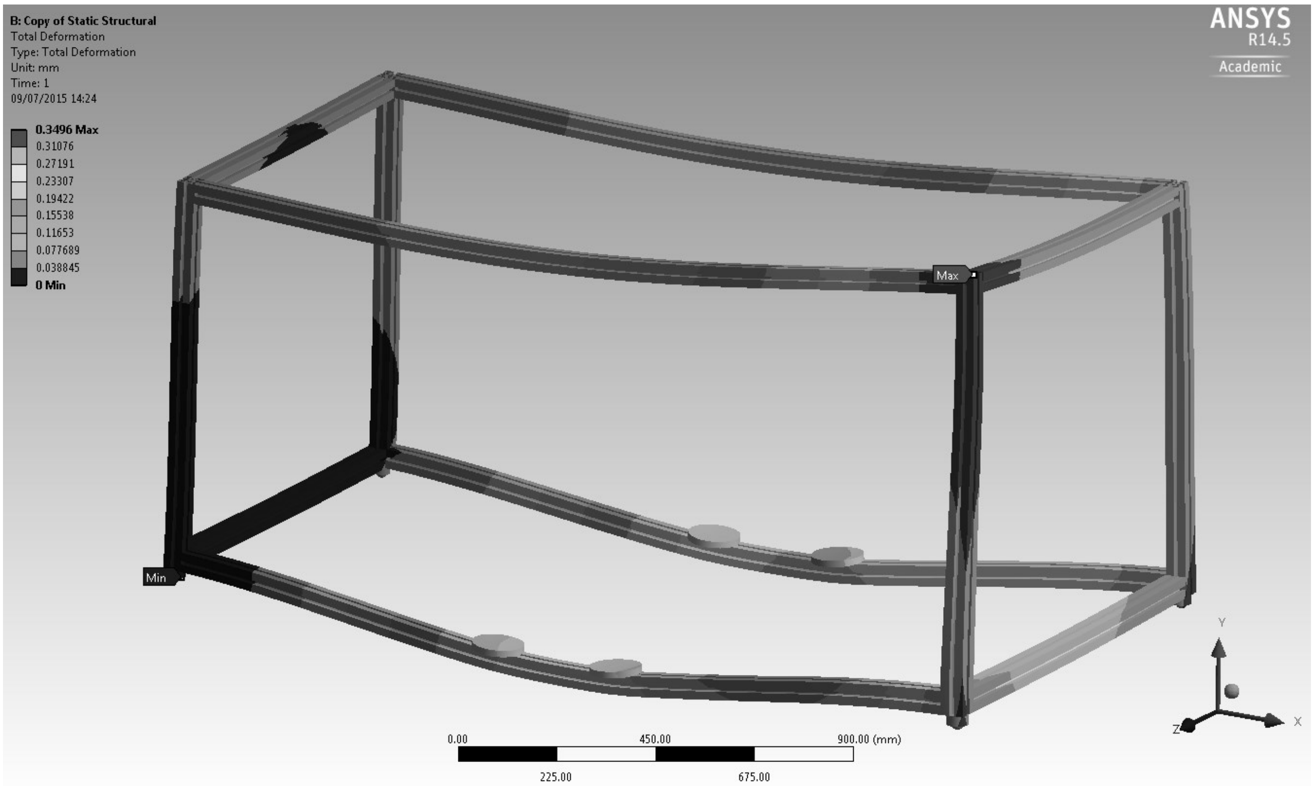


**Figure 9** Results of FE analysis of deformation of a frame structure

From the finite element results, the nodes along the extreme edges of the structure were extracted. The displacements of these were increased by a factor of 1000 to provide deformations that are visible, as seen in part 1 of figure 10. There are four edges shown along each side of the structure. The innermost of each set of four is shown as a dashed line. These are shown separately in part 2 of the figure.

The eight nodes where these innermost edges meet are used as the measurement points. The transforms taking these points from their original positions are determined by considered their new coordinates (to obtain the translation) and the new positions of neighbouring nodes (to obtain the rotation).

The interpolated transform is obtained based on these eight measurement transforms and is then applied to all the original innermost edges. The result is shown in part 3 of figure 10, and part 4 gives a comparison with how these edges transform in the finite element analysis.

The match in the comparison is poor along the two long lower edges. This is hardly surprising as the interpolated transform is based solely on what happens at the ends and these edges are doubly curved in the finite element results. Conversely however it is interesting to note that the match on the other edges is good, even though the interpolation does only involve two points from each of these edges.

## 6. CONCLUSIONS

A hybrid approach to measuring and computationally modelling components involves the prediction of the geometric model based upon the use of analysis packages to take account of factors such are thermal effects and loading.

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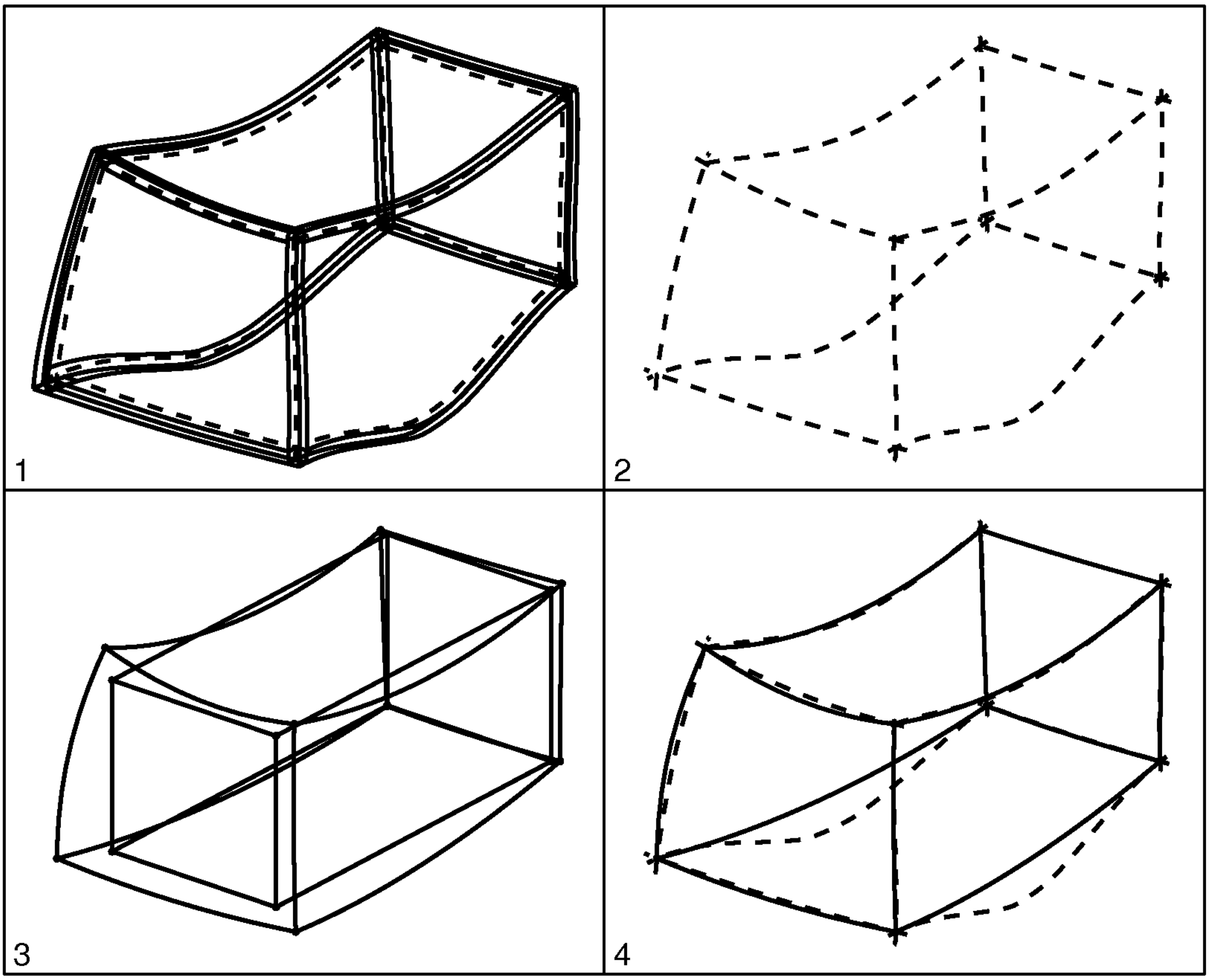


**Figure 10** Comparison of distortion from FE analysis and interpolation from corner vertices

Results from measuring the physical component (usually at discrete) points allows the predicted and actual results to be compared and discrepancies noted. This process creates a number of models of aspects of the geometry. These include: the original CAD model of the object as a full geometric model; a point-based mesh model (with many nodes) of predicted distortions derived from finite element analysis; and a point-based model of measured distortions based on few points.

The interest is in combining the information from these models. One approach is to try to modify the mesh model continuously so that it agrees with the measured results at the relevant points.

The approach for doing this has been abstracted to a situation in which the error at any measured point is represented by a rigid-body transform which maps its representation in the mesh-based model to its measured position. The methodology then requires these individual transforms to be interpolated so that a transform can be determined at any point in space. A method has been presented for doing this. It is based on considering the polyhedron formed by the points at which physical measurement takes place. This allows weights for the measured transforms to be determined based on the distance of the typical point from the geometric elements of the polyhedron.

The method has been demonstrated with some examples where the deformations are large enough that their effects can be seen. The results are encouraging. The next steps are to investigate what happens with the interpolating transform when the effects of the measured transforms are small, and then to test the method by considering geometric models and measurement of real components.

## ACKNOWLEDGMENTS

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